eigenfunctions is not an adequate criterion of correctness for the energy eigenfunctions of a nuclear model. If H_0 should be improved by perturbation theory, the lowest states would be modified by admixture of excited unperturbed states, of which the higher ones do not seem to be independent-particle states at all. As suggested by Brueckner, Eden, and Francis,[9] such admixture would probably be most obvious in high-energy phenomena of the type they discuss, in which the high-momentum part of the ground state would play the principal role.

[9] Brueckner, Eden, and Francis, Phys. Rev. 98, 1445 (1955).

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Cross Section and Angular Distributions of the (d,p) and (d,n) Reactions in C^{12} from 1.8 to 6.1 Mev*

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The reaction $C^{12}(d,p)C^{13}$ has been studied from a deuteron bombarding energy of 1.8 to 6.1 Mev. Resonances were found at 2.47, 2.67, 2.99, 3.39, 4.00, 4.6, 4.8, 5.34, and 5.64 Mev. Angular distributions of protons leaving C^{13} in the ground state show a pronounced Butler peak at 25° over the entire deuteron energy range. The angular distributions can be explained by assuming small amplitudes for compound nucleus formation interfering with large stripping amplitudes. Angular distributions of the lower energy group of protons leaving C^{13} excited to 3.09 Mev show a pronounced Butler peak at 0° and an even smaller contribution of compound nucleus formation. The reaction $C^{12}(d,n)N^{13}$ was also studied, and showed similar resonances and angular distributions. An analysis is made of the phase difference between the resonant and nonresonant parts of the cross section for the (d,p) reaction near the resonance at 4.00 Mev.

INTRODUCTION

In the last few years a great many experiments have been carried out on nuclear reactions of the (d,p) and (d,n) type which have been explained so successfully by the stripping theory of Butler.[1] In the region of deuteron energies of from 6 to 10 Mev the stripping cross section in (d,n) and (d,p) reactions appears to be very nearly the total cross section for these reactions, although in some cases there seems to be a considerable contribution of compound nucleus formation in particles observed at large angles to the direction of the incident deuterons. The purpose of the present experiments was to investigate the relative importance of the stripping reaction and compound nucleus formation in the reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,n)N^{13}$. Experiments[2] on these two reactions at energies up to 3 Mev indicated pronounced resonances which have been interpreted to be due to a large compound nucleus cross section. The object of this experiment was to investigate the excitation curves and angular distributions of the protons and neutrons produced by deuterons with energies from 1.8 Mev up to 6 Mev, in order to cover the expected transition range of energies where compound nucleus formation would become less important and stripping would become dominant. The reactions in C^{12} and the Q-values for emission of protons and neutrons are: $C^{12}(d,p)C^{13}$, $Q=2.72$ Mev; $C^{12}(d,p)C^{13*}$, $Q=-0.37$ Mev; $C^{12}(d,n)N^{13}$, $Q=-0.28$ Mev; $C^{12}(d,n)N^{13*}$, $Q=-2.65$ Mev.

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[1] S. T. Butler, Proc. Roy. Soc. (London) A208, 559 (1951).

[2] Bennett, Bonner, Hudspeth, Richards, and Watt, Phys. Rev. 59, 781 (1941); Bailey, Freier, and Williams, Phys. Rev. 73, 274 (1948); Bonner, Evans, Harris, and Phillips, Phys. Rev. 75, 1401 (1949); G. C. Phillips, Phys. Rev. 80, 164 (1950); Holmgren, Blair, Simmons, Stratton, and Stuart, Phys. Rev. 95, 1544 (1954); Takemoto, Dazai, Chiba, Ito, Suganamata, and Watanabe, J. Phys. Soc., Japan 9, 447 (1954).

EXPERIMENTAL METHOD AND RESULTS

$C^{12}(d,p)C^{13}$.—A self-supported carbon foil[3] was bombarded with deuterons from the Rice Institute 6-Mev Van de Graaff accelerator. The foil had a thickness of 155 μg/cm^2 and was oriented at an angle of 45° to the deuteron beam. The calculated energy loss of the deuterons in the foil varies from 70 kev at 2.5 Mev to 35 kev at a bombarding energy of 6 Mev. The carbon foil was at the center of a scattering chamber 5 inches in diameter that had exit ports every 10°. These ports were covered with thin aluminum foils. The protons from the nuclear reactions passed through the windows and entered scintillation counters which consisted of thin CsI crystals mounted on DuMont 6291 photo-

[3] Made by the technique described by J. D. Seagrave, Phys. Rev. 85, 197 (1952); and E. A. Milne, Phys. Rev. 93, 762 (1954).

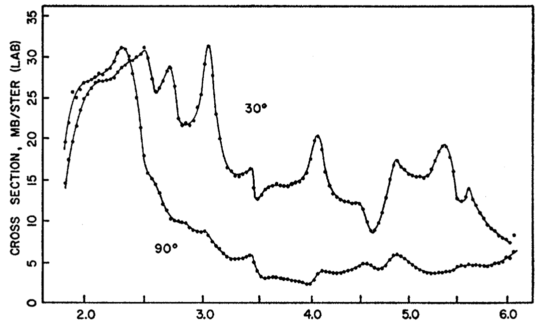

FIG. 1. Excitation function of protons from $C^{12}(d,p)C^{13}$.

multiplier tubes. Additional aluminum foils were placed between the target and the counters in order to absorb the scattered deuterons; by this method pulses from protons were made larger relative to deuteron pulses. Preliminary experiments showed that the protons emitted when C^{13} is left in the ground state had a maximum intensity near 25°, which is the expected position of the Butler stripping peak. An excitation curve from 1.8 to 6.1 Mev was obtained with one counter at 30° and a second counter at 90°. It was thought that the counts at 30° would be largely from stripping and the counts at 90° would be mainly due to compound nucleus formation. Figure 1 shows the results of this excitation curve. Pronounced resonances were obtained in both the 30° counter and the 90° counter.

In order to study the effects of the resonances on the angular distributions, experiments were made below, on, and above the 3.0 Mev resonance and these data are given in Fig. 2. Figure 3 shows similar distributions near the 4.0-Mev resonance taken at intervals of 10°. Figure 4 shows the angular distribution at 4.5 Mev. At a deuteron energy of 4.75 Mev the second group of protons leaving C^{13} in its first excited state at 3.09 Mev have sufficient energy to be separated from the scattered deuterons. A pulse-height distribution curve which was obtained at this bombarding energy is shown in Fig. 5. The two groups of protons, P_0 and P_1, are

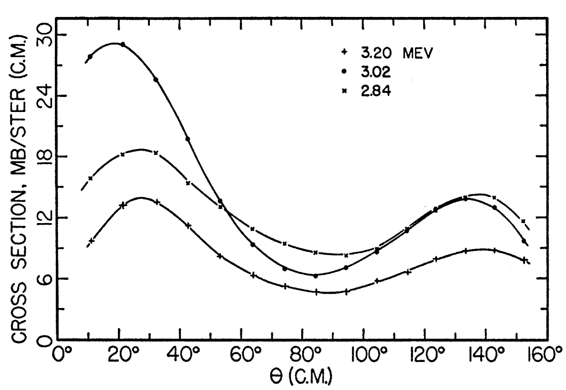

FIG. 2. Angular distribution of protons near the 2.99-Mev resonance.

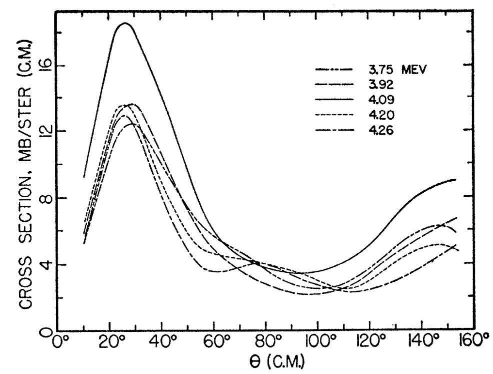

FIG. 3. Angular distribution of protons near the 4.00-Mev resonance.

resolved and the angular distribution of both groups of protons was obtained. The angular results are shown in Fig. 6. The long-range group P_0 shows a similar distribution to those obtained at lower bombarding energies while the other group P_1 is strongly peaked at 0° as would be expected from the stripping reaction.

Additional data on angular distributions were taken at twelve angles of observation with deuteron energies of from 3.23 to 4.37 Mev. The results are given in Fig. 7.

$C^{12}(d,n)N^{13}$.—An excitation curve for the neutrons from the (d,n) reaction was observed at 0° using a modified long counter[4] to detect the neutrons. This excitation curve from a deuteron energy of 2.7 to 5.0 Mev is shown in Fig. 8. Above a deuteron energy of 3.09 Mev there are two neutron groups which are counted by the long counter with nearly equal efficiencies. The excitation curve for neutrons shows several resonances and is similar to that obtained for protons. The absolute cross section for production of neutrons was measured at 3.64 Mev by comparing the counting rate from the target in a true long counter with that

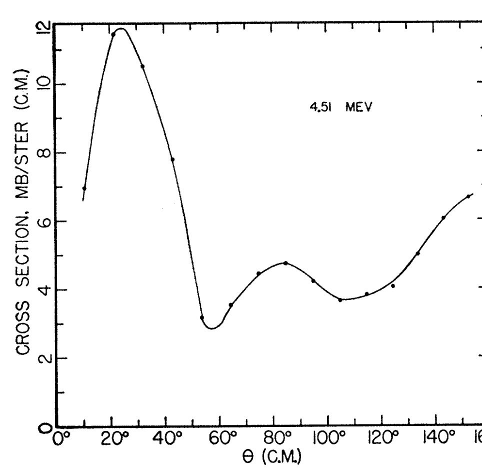

FIG. 4. Angular distribution of protons at a deuteron energy of 4.51 Mev.

[4] Brugger, Bonner, and Marion, Phys. Rev. **100**, 84 (1955).

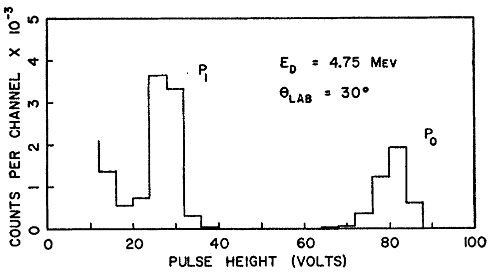

FIG. 5. A pulse-height distribution histogram showing the two proton groups P_0 and P_1.

from a Ra-Be source which had been calibrated by the Bureau of Standards. The measured cross section in the laboratory coordinate system is 45 mb/sterad at 0°.

Angular distributions of the neutrons were obtained with 3.58-, 3.98-, and 4.28-Mev deuterons which covered the region of the 4.0-Mev resonance. Two types of experiments were carried out. The first method made use of the modified long counter as a detector and so counted both groups of neutrons. These data are given by the upper curves of Fig. 9. These curves are strongly peaked at 0° at all three energies. Other experiments were carried out with an energy-sensitive neutron detector[5] which was made up of four spherical anthracene scintillators, each with a diameter of 2.5 mm. The discriminator was set so that only the high-energy group of neutrons was counted. These data are given by the lower curve of Fig. 9. No calibration of the relative sensitivities of the two different counters was carried out and so the relative ordinates of the two curves are arbitrary. The high-energy neutron group has a maximum intensity at about 20° which is expected from stripping theory. A comparison of the two distributions indicates that the lower-energy neutron group is strongly peaked at 0° as is the lower energy proton group.

From the shapes of the two angular distribution curves it is estimated that the cross section for high-energy neutron production at 30° with 3.64-Mev deuterons is approximately the same as the cross section (14 mb/sterad) for high-energy protons.

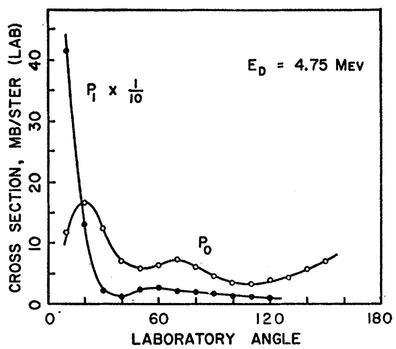

FIG. 6. Angular distribution of protons (P_0) leaving C^{13} in the ground state and protons (P_1) leaving C^{13} in the excited state at 3.09 Mev.

[5] Taylor, Lönsjö, and Bonner, Phys. Rev. **100**, 174 (1955).

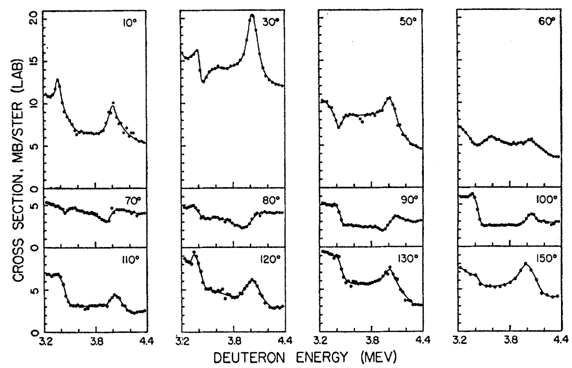

FIG. 7. Excitation functions of $C^{12}(d,p)C^{13}$ from a deuteron energy of 3.23 Mev to 4.37 Mev at various angles of observation.

DISCUSSION OF RESULTS

Table I gives a list of resonances found from the data of Fig. 1 for the (d,p) reaction. After correction is made for target thickness an energy can be assigned to the resonances. Table I also gives the excitation energy of N^{14} at each resonance observed in the present experiment as well as the excitation energies obtained from other experiments. A number of levels which have been observed in other experiments are not apparent in our data. Failure to observe some of these levels is undoubtedly due to the thickness of the target, which was 35 to 70 kev depending on the energy of the deuterons; this would not allow the separation of resonances with an energy difference of less than this amount. Other levels were probably missed because their relative intensity was less than that of the observed levels.

Table II gives a list of the resonances in the $C^{12}(d,n)N^{13}$ excitation function obtained from Fig. 8. The accuracy of the energy determination of the (d,n) resonances was considerably less than for the (d,p) resonances as a proton moment detector was not used to measure accurately the magnetic field in the magnetic analyzer. For this reason the accuracy of the level determinations is about ± 30 kev.

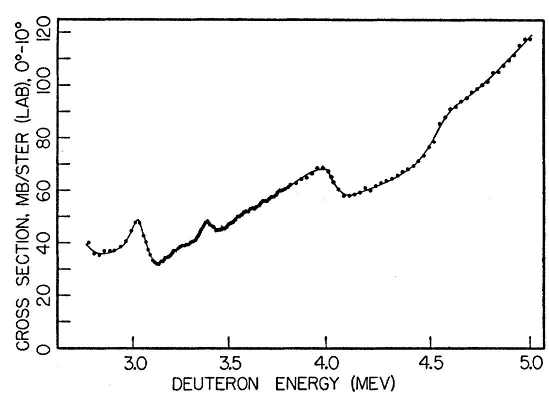

FIG. 8. Excitation function of all neutrons from $C^{12}(d,n)N^{13}$ at 0°.

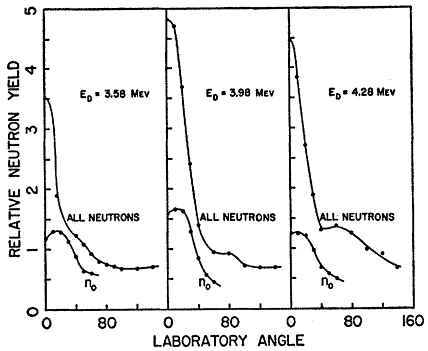

FIG. 9. Angular distribution of neutrons at a deuteron energy Of 3.58, 3.98, and 4.28 Mev. The upper curves give the angular distribution of both groups of neutrons and the lower curves give the angular distribution of the high-energy group of neutrons.

Many of the levels found in the $C^{12}(d,p)C^{13}$ reaction agree with those found in the reactions $B^{10}(\alpha,p\gamma)C^{13}$ and $B^{10}(\alpha,n)N^{13}$ as well as those in the $C^{12}(d,n)N^{13}$ reaction. It seems clear that these must be formed by compound nucleus formation.

The angular distribution of the protons and neutrons leaving C^{13} and N^{13} in the ground state show a peak near 25° that corresponds rather well in position with those predicted by Butler's analysis of stripping reactions. From the spins of the target and final nuclei one expects $l=1$ particles to be captured. At larger angles the intensity does not go to zero as predicted by the simple stripping theory; however, zero intensities are eliminated when Coulomb forces are included in the stripping theory.[6] The large cross sections near 140° are probably due to compound nucleus formation as they are much larger than predicted by the stripping theory in which the Coulomb effects are considered.

TABLE I. Observed resonances in $C^{12}(d,p)C^{13}$ after correction for target thickness.

Deuteron energy (Mev)	Levels in N^{14} Present experiment (Mev)	Other data[a] (Mev)	Level width[a] (Mev)	Angular momentum and parity[a]
2.465	12.38	12.42	43	4−
2.669		12.50	36	
	12.55			
		12.61	50	
		12.69	14	3−
		12.79	14	4+
2.992	12.83	12.82	5	4−
		12.92	21	4+
3.388	13.17	13.16	20	
		13.24	140	
4.004	13.70	13.72	150[b]	
4.6			broad	
4.8			broad	
5.336	14.84		200	
5.635	15.09		350	

[a] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 126 (1955), and the data of Bonner, Kraus, Marion, and Schiffer, Phys. Rev. (to be published) on the reactions $B^{10}(\alpha,p\gamma)C^{13}$ and $B^{10}(\alpha,n)N^{13}$.
[b] Width determined by this experiment.

[6] W. Tobocman, Phys. Rev. **94**, 1655 (1954); W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).

The resonances at 2.99, 4.00, and 5.34 Mev are more pronounced at the 25° angle of the Butler peak than at 90°; this seems to contradict the idea that compound nucleus formation is the cause of these resonances. However, this phenomenon can be explained even if the compound nucleus formation at these resonances is only a few percent of the magnitude of stripping if it has a symmetrical angular distribution which has a minimum at 90° and a maximum near 0° and 180°. Under these conditions more pronounced resonances would be observed at 25° than at 90°.

The angular distribution of the low-energy protons and neutrons leaving C^{13} and N^{13} in their first excited states, peak at 0° as is expected on the Butler theory where the captured particle is expected to have $l=0$. The data on the angular distribution of low-energy protons which are shown in Fig. 6 agrees quite well with the Butler stripping theory even at angles up to 120°. This indicates that most of the cross section is due to stripping and there is very little contribution from compound nucleus formation. This result is expected from a classical picture of the stripping reaction where the disintegration particle is expected to have about the same velocity as the incident deuteron. Thus the stripping reaction at a deuteron energy of 4.75 Mev is less likely to give a 6.8-Mev proton than a 3.7-Mev proton.

TABLE II. Resonances in $C^{12}(d,n)N^{13}$ observed at 0° after corrections for target thickness.

Deuteron energy (Mev)	Levels in N^{14} (Mev)
3.01	12.84
3.36	13.14
3.95	13.65
4.61	14.21

Another indication that the low-energy groups of protons and neutrons are produced more from the stripping reaction and less from compound nucleus formation is obtained by comparing the total neutron excitation curve of Fig. 8 with the high-energy proton curve of Fig. 1. The resonances at 4.0 and 4.6 Mev are not as pronounced in the neutron curve where a considerable contribution at 0° is due to the lower energy group of neutrons.

The observation of the resonances at a series of different angles is shown in Fig. 7. These resonances exhibit a variety of interesting effects. The 30° curve shows an "ordinary" resonance at 4.0 Mev and an "unsymmetrical" resonance at 3.4 Mev. The 50° and 60° curves have "antiresonances" at 3.4 Mev. The 90° curve has an "unsymmetrical" resonance at 4.0 Mev but reversed from that at 30° and 3.4 Mev. These interference phenomena suggest that one can measure a phase difference between the resonant and nonresonant parts of the cross section. This can be done in a simple way as follows: let the cross section be the square of an amplitude that has two parts, a nonresonant part, $\sigma_B^{\frac{1}{2}}$, and a resonant part which obeys the Breit-Wigner formula. If δ is the phase difference between these two components, one can write the cross section as

$$\sigma(E,\theta)=\left|\sigma_B^{\frac{1}{2}}(E,\theta)e^{i\delta(\theta)}+\tfrac{1}{2}i\Gamma\frac{[\sigma_R(\theta)]^{\frac{1}{2}}}{(E-E_R)+\frac{1}{2}i\Gamma}\right|^2, \quad \text{(I)}$$

where Γ is the width of the resonance, E the deuteron energy, θ the angle of observation in the center-of-mass system, and E_R is the resonance energy. The amplitude $\sigma_B^{\frac{1}{2}}$ is determined by estimating what the cross section would be if the resonance were not present. At 4.0 Mev, $\theta(\text{lab})=30°$, this was estimated to be 63% of the total cross section of the remaining 37%, the calculated value of the interference term between the resonant and nonresonant parts is 33% and only 4% is due to the resonant term alone. The contribution to the experimental cross section of the resonance term varies with angle from 1 to 8% for this resonance. Only at $\theta=64°$ is the contribution less than 3%. Values of $[\sigma_R(\theta)]^{\frac{1}{2}}$ and $\delta(\theta)$ are chosen

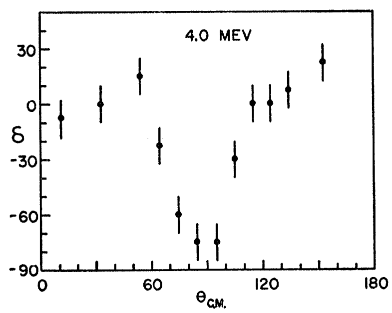

FIG. 10. The quantity $\delta(\theta)$ as a function of the angle of observation for the 4.0-Mev resonance.

at each angle so that the calculated cross section fits the experimental points. The quantity $\delta(\theta)$ is shown in Fig. 10 for the 4.00-Mev resonance. The observed resonance width of 150 kev was used in the calculations. The phase shift $\delta(\theta)$ appears to be quite small except for angles between 60° and 110°, in which region there is a rapid change with angle. It is in just this region that the experimental cross section assumes its lowest values; however, in the range from 60° to 110°, the resonance contribution averages 4% of the experimental cross section. The curve is fairly symmetrical about 90° in the center-of-mass system. If the assumed background cross section is altered by 10%, no appreciable change in the shape of $\delta(\theta)$ curve results.

One would expect $|[\sigma_R(\theta)]^{\frac{1}{2}}|^2$ to be the cross section if the background were not present. The experimentally

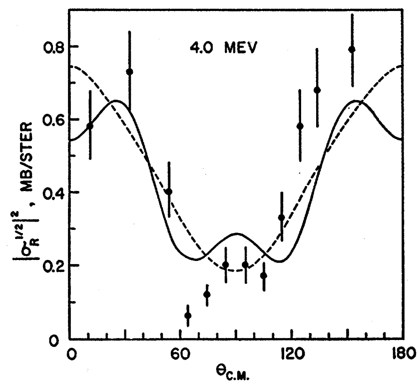

FIG. 11. The quantity $|[\sigma_R(\theta)]^{\frac{1}{2}}|^2$ as a function of the angle of observation for the 4.0-Mev resonance. The solid curve gives the angular distribution calculated for $l_d=2$, $J=3^+$, $l_p=3$, and channel spin$=0$; the dashed curve gives the distribution for $l_d=1$, $J=2^-$, $l_p=2$, and channel spin$=0$.

determined values of this quantity are shown in Fig. 11 for the 4.00-Mev resonance. They are approximately symmetrical about 90° as would be expected for pure compound nucleus formation. An attempt was made to fit this curve for various values of the angular momenta involved. The two calculated angular distributions which best approximate the experimental points are also shown in Fig. 11. The dashed curve is that calculated for a compound nucleus state of 2^- and the solid curve is that for a 3^+ state. Each calculated curve represents the distribution for only one possible choice of the l-values and channel spin. The other possibilities give poorer fits. The angular distributions calculated for J-values of 2^- and 3^+ both approximately fit the experimental distribution. However, the experimental minima near 60° and 110° are only shown by the 3^+ distribution. Since J-values of 4 have been assigned to the lower energy resonances which are considerably narrower (see Table I) than the 150 kev observed for the 4.00-Mev resonance, J-values less than 4 are to be expected for this compound nucleus state. Therefore, it seems likely that the 13.70-Mev state of N^{14} which is formed at the 4.00-Mev resonance has a J-value of 3.

The agreement is probably as good as one might expect on the basis of such a simplified theory. A more refined theory might write the cross section as an incoherent sum over magnetic quantum numbers and take into account the fact that the nonresonant part has contributions from deuterons of many different angular momenta. This procedure would introduce so many arbitrary parameters that a fit of the experimental data would become meaningless.